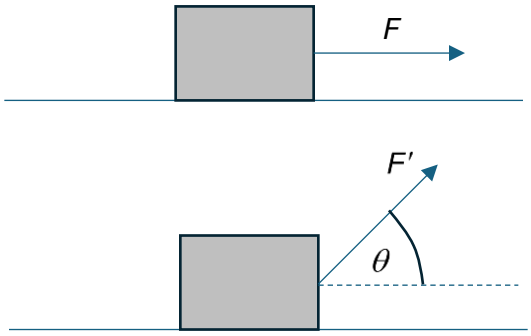
Teacher notes
Topic A

A little friction problem

A block of mass *m* is pushed along a horizontal rough surface with a horizontal force of magnitude *F*. The block slides with constant speed. A force F' acts on an identical block, on the same surface. This force makes an angle θ to the horizontal. This block also moves at constant speed.



The kinetic friction coefficient is $\mu = 0.50$.

(a) For $\theta = 45^\circ$ what is $\frac{F'}{F}$?

(b) For what angle θ is this ratio minimum? (Use your GDC.)

(c) For which angle θ (other than 0) is the ratio equal to 1?

Answer

(a) Since both blocks move at constant speed,

$$F = \mu N = \mu mg$$

and

$$F'\cos\theta = \mu N'$$
$$F'\sin\theta + N' = mg \Rightarrow N' = mg - F'\sin\theta$$

Hence,

$$F'\cos\theta = \mu(mg - F'\sin\theta)$$
$$F'(\cos\theta + \mu\sin\theta) = \mu mg$$
$$F' = \frac{\mu mg}{\cos\theta + \mu\sin\theta}$$
$$F' = \frac{F}{\cos\theta + \mu\sin\theta}$$

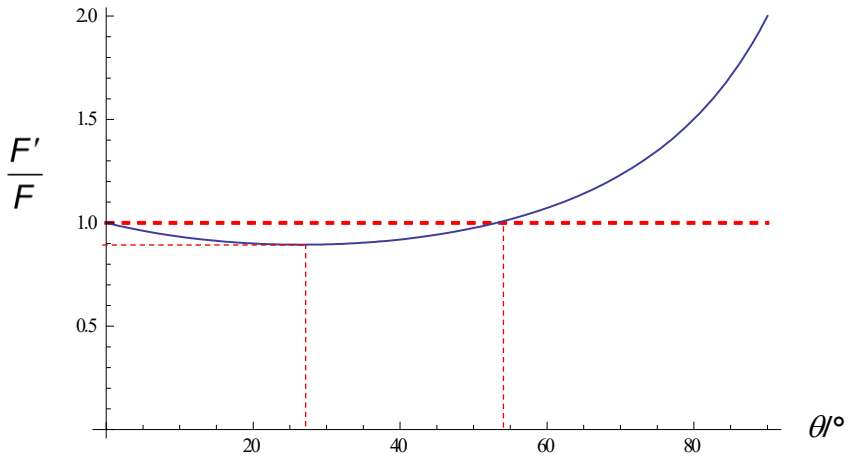
And finally:

$$\frac{F'}{F} = \frac{1}{\cos\theta + \mu\sin\theta}.$$

For $\theta = 45^\circ$,

$$\frac{F'}{F} = \frac{1}{\frac{\sqrt{2}}{2} + \frac{1}{2}\times\frac{\sqrt{2}}{2}}$$
$$= \frac{4}{3\sqrt{2}}$$
$$= \frac{2\sqrt{2}}{3} \approx 0.94$$

(b) To avoid calculus, we just plot $\frac{F'}{F} = \frac{1}{\cos\theta + \frac{1}{2}\times\sin\theta}$ against θ:



The minimum is at about $\theta = 27°$. The minimum value is about 0.89.

(c) From the graph the ratio is 1 at about $\theta = 53°$.